CHAPTER 9

RESISTANCE (PART 1)

Provided that the temperature of a material remains constant, the resistance of that material will depend on three factors. These are -

(a) the type of the material
(b) the length of the material
(c) the cross sectional area of the material

9.1 TYPE OF MATERIAL

It has been stated that, due to their atomic structures, gold, silver and copper vary in their abilities to act as conductors (Chapter 2). This inbuilt factor affecting a material's resistance is known as resistivity and has the symbol rho (ρ). The resistivity of a material is measured between opposite faces of a cube of that material, each side of the cube having a length of one metre. The unit for resistivity is the ohm metre (Ω m).

Typical values of resistivity are given in the following table.

MATERIAL	RESISTIVITY (OHM METRE)
Silver	1.66×10^{-8}
Copper	1.72×10^{-8}
Aluminium	2.8×10^{-8}
Nickel	13.6×10^{-8}
Manganin	48×10^{-8}
Nichrome	110×10^{-8}

As the value of resistivity of materials increases so does the resistance of the materials increase. Resistance is said to be directly proportional to resistivity.

$$R \propto \rho$$

9.2 LENGTH (l)

Any conductor of fixed length and cross-sectional area contains a certain number of atoms. An electron under the influence of an e.m.f. moves from one end of the conductor to the other end of the conductor, colliding with the atoms in the conductor in the process. If the cross-sectional area of the conductor is kept constant while the length of the conductor is increased, the travelling electron would find more atoms in its path as it moved through the conductor. This means the opposition to the movement of the electron is increased. Opposition to the movement of electrons is called resistance, so it can be seen that an increase in the length of a conductor (all other factors remaining constant) will cause an increase in the resistance of the conductor. Resistance is said to be directly proportional to the length of a conductor.

$$R \propto l$$

9.3 CROSS-SECTIONAL AREA (A)

Increasing the cross-sectional area of a given length of conductor material increases the number of electrons free to move within the material. The application of a potential difference to the conductor of larger C.S.A. will cause a greater number of electrons to move in the conductor, or the current would increase. For the current to increase for a fixed potential difference the resistance must decrease. Resistance is said to be inversely proportional to cross-sectional area.

$$R \propto \frac{1}{A}$$

Combining the three factors controlling resistance in the correct proportionalities gives the equation —

$$R = \frac{\rho l}{A}$$

Where -

R is in ohms
ρ is in ohm metres
l is in metres
A is in metres squared

Example 9.1

Calculate the resistance of 100 metres of copper conductor that has a cross-sectional area of 12 millimetres squared.

$\rho = 1.72 \times 10^{-8}\ \Omega\,m$

$l = 100\ m$

$A = 12 \times 10^{-6}\ m^2$

$R = ?\ \Omega$

$$R = \frac{\rho l}{A}$$

$$= \frac{1.72 \times 10^{-8} \times 100}{12 \times 10^{-6}}$$

$$= 0.143\ \Omega$$

Example 9.2

Calculate the length of nichrome wire required to make a resistor of 220 ohms if the c.s.a. of the wire is 10 mm^2.

$R = 220\ \Omega$

$A = 10 \times 10^{-6} m^2$

$\rho = 110 \times 10^{-8}\ \Omega\,m$

$l = ?m$

$$R = \frac{\rho l}{A}$$

$$l = \frac{RA}{\rho}$$

$$= \frac{220 \times 10 \times 10^{-6}}{110 \times 10^{-8}}$$

$$= 2000\ m$$

9.4 VARIATIONS OF RESISTANCE due to variations in length or cross-sectional area when the same material is used in the resistor.

(a) Length (l)

Resistance of a conductor of fixed c.s.a. increases with the increase in length of the conductor. If the length of the conductor is doubled, the resistance is doubled, indicating that the ratio of the initial resistance to the final resistance is equal to the ratio of the initial length to the final length.

If -

R_1 = initial resistance
R_2 = final resistance
l_1 = initial length
l_2 = final length

Then - $$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

Transposing gives — $$R_2 = \frac{R_1\ l_2}{l_1}$$

Example 9.3

A 100 metre length of resistance wire has a resistance of 12 ohms. Calculate the resistance of 250 metres of the same wire.

$R_1 = 12\ \Omega$

$l_1 = 100\ m$

$l_2 = 250\ m$

$R_2 = ?\ \Omega$

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

$$R_2 = \frac{R_1\ l_2}{l_1}$$

$$= \frac{12 \times 250}{100}$$

$$= 30\ \Omega$$

Example 9.4

A length of copper wire had a resistance of 150 ohms. When 50 metre of the wire was measured with an ohmmeter its resistance was found to be 60 ohms. How long was the origional piece of wire.

$R_1 = 150\ \Omega$

$R_2 = 60\ \Omega$

$l_1 = ?\ m$

$l_2 = 50\ m$

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

$$R_1 l_2 = R_2 l_1$$

$$l_1 = \frac{R_1 l_2}{R_2}$$

$$= \frac{150 \times 50}{60}$$

$$= 125\ m$$

(b) CROSS-SECTIONAL AREA (A)

Resistance is inversely proportional to cross-sectional area. If the cross-sectional area of a conductor is doubled, the conductor resistance is halved. This function is written mathematically as -

$$\frac{R_1}{R_2} = \frac{\frac{1}{A_1}}{\frac{1}{A_2}}$$

$$\frac{R_1}{R_2} = \frac{A_2}{A_1}$$

and transposing for R_2 gives

$$R_2 = \frac{R_1 A_1}{A_2}$$

Example 9.5

A length of nichrome resistance wire has a resistance of 100 ohms and a cross-sectional area of 12 millimetres squared. Determine the resistance of the same length of nichrome wire whose cross-sectional area is 15 millimetres squared.

$R_1 = 100\ \Omega$

$A_1 = 12 \times 10^{-6}\ m^2$

$A_2 = 15 \times 10^{-6}\ m^2$

$R_2 = ?\ \Omega$

$$\frac{R_1}{R_2} = \frac{A_2}{A_1}$$

$$R_2 = \frac{R_1 A_1}{A_2}$$

$$= \frac{100 \times 12 \times 10^{-6}}{15 \times 10^{-6}}$$

$$= 80\ \Omega$$

Example 9.6

A rectangular copper conductor whose cross sectional dimensions are 30 mm x 5 mm has a resistance of 2 ohms. Calculate the resistance of the same length of conductor material whose c.s.a. dimensions are 20 mm x 15 mm.

$R_1 = 2\,\Omega$

$R_2 = ?\,\Omega$

$A_1 = 30 \times 10^{-3} \times 5 \times 10^{-3}\,m^2$

$A_2 = 20 \times 10^{-3} \times 15 \times 10^{-3}\,m^2$

$$\frac{R_1}{R_2} = \frac{A_2}{A_1}$$

$$R_2 = \frac{R_1\,A_1}{A_2}$$

$$= \frac{2 \times 30 \times 10^{-3} \times 5 \times 10^{-3}}{20 \times 10^{-3} \times 15 \times 10^{-3}}$$

$$= 1\,\Omega$$

(c) Circular conductors

The area of a circle is given by the equation -

$$A = \frac{\pi d^2}{4}$$

Substituting in $\frac{R_1}{R_2} = \frac{A_2}{A_1}$

Gives $$\frac{R_1}{R_2} = \frac{\frac{\pi d_2^2}{4}}{\frac{\pi d_1^2}{4}}$$

Cancelling the $\frac{\pi}{4}$

Gives $$\frac{R_1}{R_2} = \frac{d_2^2}{d_1^2}$$

which means that in a circular conductor the resistance of the conductor is inversely proportional to the square of the diameter of the conductor.

Example 9.7

A length of copper wire has a diameter of 2 millimetres and a resistance of 50 ohms. Calculate the resistance of the same length of copper wire if the diameter of the wire is 4 millimetres.

$R_1 = 50\,\Omega$

$d_1 = 2 \times 10^{-3}$ m

$d_2 = 4 \times 10^{-3}$ m

$R_2 = ?\,\Omega$

$$\frac{R_1}{R_2} = \frac{d_2^2}{d_1^2}$$

$$R_2 = \frac{R_1\,d_1^2}{d_2^2}$$

$$= \frac{50 \times 2 \times 2 \times 10^{-6}}{4 \times 4 \times 10^{-6}}$$

$$= 12.5\,\Omega$$

9.5 SIMULTANEOUS variations in length and area.

Combining the proportionalities of resistance, length and area variations, results in the equation

$$\frac{R_1}{R_2} = \frac{l_1\,A_2}{l_2\,A_1}$$

Transposing for R_2 gives

$$R_2 = \frac{R_1\,l_2\,A_1}{l_1\,A_2}$$

Example 9.8

A piece of nichrome resistance wire has a resistance of 100 ohms, a length of 50 metres and cross-sectional area of 10 millimetres squared. Calculate the resistance of a piece of nichrome wire, 200 metres in length that has a cross-sectional area of 5 millimetres squared.

$$R_1 = 100\ \Omega$$
$$l_1 = 50\ \text{m}$$
$$A_1 = 10 \times 10^{-6}\ \text{m}^2$$
$$l_2 = 200\ \text{m}$$
$$A_2 = 5 \times 10^{-6}\ \text{m}^2$$
$$R_2 = ?\ \Omega$$

$$\frac{R_1}{R_2} = \frac{l_1 A_2}{l_2 A_1}$$

$$R_2 = \frac{R_1 l_2 A_1}{l_1 A_2}$$

$$= \frac{100 \times 200 \times 10 \times 10^{-6}}{50 \times 5 \times 10^{-6}}$$

$$= 800\ \Omega$$

Example 9.9

A 100 m length of nichrome wire was found to have a resistance of 150 ohms. Determine the resistance of a length of wire of the same material, but half the length and four times the cross sectional area.

$$R_1 = 150\ \Omega$$
$$R_2 = ?\ \Omega$$
$$l_1 = 100\ \text{m}$$
$$l_2 = 50\ \text{m}$$
$$A_1 = A_1\ \text{m}^2$$
$$A_2 = 4A_1\ \text{m}^2$$

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{A_2}{A_1}$$

$$R_1 l_2 A_1 = R_2 l_1 A_2$$

$$R_2 = \frac{R_1 l_2 A_1}{l_1 A_2}$$

$$= \frac{150 \times 50 \times A_1}{100 \times 4A_1}$$

$$= 18.75\ \Omega$$

Equations in this chapter

(1) $R = \dfrac{\rho l}{A}$

(2) $R_2 = \dfrac{R_1 l_2}{l_1}$

(3) $R_2 = \dfrac{R_1 A_1}{A_2}$

(4) $R_2 = \dfrac{R_1 l_2 A_1}{l_1 A_2}$

(5) $R_2 = \dfrac{R_1 (d_1)^2}{(d_2)^2}$

TUTORIAL 1.9

(1) Calculate the resistance of 100 metres of copper wire whose c.s.a. is 5 mm^2 and whose resistivity is 1.72×10^{-8} ohm metre.

(2) Calculate the length of aluminium conductor ($\rho = 2.8 \times 10^{-8}$) whose c.s.a. is 4 mm^2 that would have a resistance of 1.4 ohms.

(3) A length of copper conductor was found to have a resistance of 50 ohms. Tests on 10 metres of the same type and size conductor gave a resistance reading of 7.5 ohms. Calculate the length of the origional conductor.

(4) A conductor whose c.s.a. is 7.5 mm^2 has a resistance of 20 ohms. Calculate the c.s.a. of a conductor of the same length and material which has a resistance of 12.5 ohms.

(5) A conductor 2 metre long with a c.s.a. of 1 square millimeter has a resistance of 0.017 ohms. Determine the resistance of 25 metres of the same material whose c.s.a. is 0.25 mm^2.